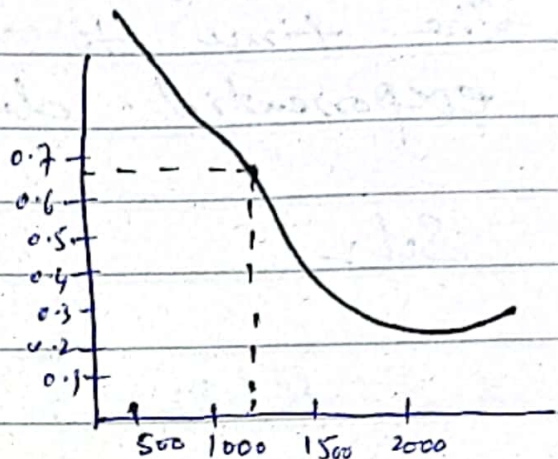Q No

We have tested 20x unit and found that our MTBF is 2996 Hrs what is the reliability of product @ 1200 hr of operation.

Data:.

$$t = 1200$$
$$\theta = 2996$$

We know that

$$MTTF = \theta = \frac{\text{operation Time}}{\text{No. of failure}}$$

$$MTBF = \lambda = \frac{\text{No. of Failure}}{\text{operation time}}$$

$$\theta = \frac{1}{\lambda} \quad \Rightarrow \quad \lambda = \frac{1}{\theta}$$

Reliability $= R(t) = e^{-\lambda t}$

$$R(t) = e^{-\left(\frac{t}{\theta}\right)}$$

$$= e^{-\left(\frac{1200}{2996}\right)}$$

$$R(t) = 0.67 \cong 67\%$$

failure rate $= F = 1 - R.$

$$= 1 - 67\%$$
$$F = 33\% \quad (\text{un reliable rate}$$

QNo 2

what is the hightst Faiture rate for
a product 2f it is to have an
probability of Servival (i-e Succerful
operation) of 95% at 4000 Hr ? Assume
the time to failure following an of
exponential distribution:

Sol:-

$$R(t) = e^{-\lambda t}$$

$$0.95 = e^{-[4000\,\lambda]}$$

takiy ln on both Side

$$\ln (0.95) = \ln e^{-[4000\,\lambda]}$$

$$-0.0512 = -4000 \times \lambda \times \ln e$$

$$\ln e = 1$$

$$-0.0512 = -4000\,\lambda$$

$$\lambda = \frac{-0.0512}{-4000}$$

$$\lambda = 12.8 \times 10^{-6}$$

Example: 20 units are put on test & run at their normal operating condition for 1000 hrs if 6 of these units fail at the following hours

(550, 480, 680, 790, 860, 620)

what is the mean time to failure of the product:

Solution:-

$$\text{"n Reliability} = f = \frac{6}{20} = 0.3 \cong 30\%$$

$$\text{Reliability}(R) = 1 - 0.3 = 0.70 \cong 70\%$$

$$MTTF = \theta = \frac{(1000 \times 14) + 550 + 480 + 680 + 790 + 860 + 620}{6}$$

$$\theta = \frac{17980}{6} = 2996 \text{ Hrs to failure}$$

$$\theta = \frac{1}{\lambda} = \lambda = \frac{1}{\theta} = \frac{1}{2996}$$

$$\lambda = 0.00033$$

Weibull distribution:

Q No:

Data:-

$$t = 5000$$
$$\theta = 8000$$
$$R(t) = ??$$
$$\beta = 2$$

$$\text{Reliability} = R(t) = e^{-\left(\frac{5000}{8000}\right)^2}$$

$$= e^{-0.3906}$$

$$R(5000) = 0.676 \cong 67\%$$

Imported:

Final Exam 2024:

Problem:- twenty components will CFR
constant Failure rate) was observed being
used in a height stressed environment
After 25hr of use Seven of then failure
at time in hr.

2.1, 8.3, 10.9, 15.2, 16.3, 20.5, 23.8
while the remaing still functioning. Calculate the
following

ⓐ MTTF

② Failure rate

③ the time which reliability 95%.

④ the Reliability of 50 Hrs of use.

① $MTTF = \dfrac{(13\times25)+2.1+8.3+10.9+15.2+16.3+20.5+23.8}{7}$

$= \dfrac{422.1}{7} = 60.3 \ hrs/failure$

② Failure rate:

$$\lambda = \frac{1}{\theta} = \frac{1}{60.3}$$

$$= 0.0166 \ Failure/hrs.$$

(B)(3)

Time at which reliability is 95%

$$R(t) = e^{-\lambda t}$$

$$0.95 = e^{-0.0166t}$$

taking $\ln$ on both side

$$\ln(0.95) = -[0.0166 \times t] \cdot \ln e$$

$\ln e = 1$

$$-0.0512 = -0.0166 \times t$$

$$t = \frac{0.0512}{0.0166}$$

$$\boxed{t = 3.09 \text{ hrs}}$$

④ Reliability after 50 hrs of use

$$R(t) = e^{\lambda t}$$

$$R(50) = e^{-0.0166 \times 50}$$

$$R(50) = e^{-0.83} = 0.436$$

$$\boxed{R(50) = 43.6\%}$$

Q.No:

A Six Sided die is tossed

(a) what is the probability of getting a "2"
(b) ~ ~ ~ number that is almost "4"
(c) ~ ~ ~ 3 or 5
(d) ~ ~ ~ No. that is greater then "3"
(e) ~ ~ ~ getting No. that is less then or equal to 5"

Solution:-

$$S \cdot S = \{1, 2, 3, 4, 5, 6\}.$$

(a) Probability getting "2"

$$P(2) = \frac{1}{6} = 0.166 \simeq 16.6\%$$

(b) Probability No. that is almost "4"

$$S \cdot S = \{1, 2, 3, 4\}$$

$$P(x \leq 4) = \frac{4}{6} = \frac{2}{3} = 0.666 \simeq 66.66\%$$

(c) Probability 3 or 5

$$P(3 \text{ or } 5) = \frac{2}{6} = \frac{1}{3} = 0.333 \simeq 33.3\%$$

(d) Probability of greater then 3

$$\{4, 5, 6\}.$$

$$P(x > 3) = \frac{3}{6} = \frac{1}{2} = 0.5 = 50\%$$

(e) less then equal 5

$$S.S = \{1, 2, 3, 4, 5, 6\}.$$

$$P(x \leq 5) = \frac{5}{6} = 0.83 = 83\%$$

Q.No.

A Jar contains 7 Red marbles 6 green marbles 5 blue marble and 2 yellow marbles:-

① what is the Probability of Selecting green marbles.

② " " " " " " blue marble

③ Green or Yellow

④ Red then blue with replacement

⑤ Red then blue without replacement

⑥ what is the probability of Selecting Red then green marble or getting a blue marble and then yellow marble with replacement

Sol:-

Color		marble Qty
Red (R) =		7
Green (G) =		6
Blue (B) =		5
Yellow (Y) =		2
Total =		20

① Probability of green marble

$$P(G) = \frac{6}{20} = \frac{3}{10} = 0.3 = 30\%$$

Not Selecting the green marble

$$= 1 - 0.3 = 0.70 = 70\%$$

② Probability of Selecting blue marble

$$P(B) = \frac{5}{20} = \frac{1}{4} = 0.25 \simeq 25\%$$

③ Probability of Selecting Green & Yellow

$$P(G) + P(Y)$$

$$\frac{6}{20} + \frac{2}{20} = \frac{8}{20} = \frac{2}{5} = 0.4 = 40\%$$

There are 40% chance of Selecting
B or yellow marbles.

④ Red then blue with replacement it
means once you take out red marble
on 1st try you put it back for
2nd try

$$= P(R) \cdot P(B).$$

P(R then Blue) = P(R) · P(B)

$$= \frac{7}{20} \times \frac{5}{20} = 0.0875 \simeq 8.75\%$$

⑤ Red then blue without replacement its
mean once you take out red marble
on 1st try you put separat (other basket)
and then try.

$$P(R) \cdot P(b)$$

$$= \frac{7}{20} \cdot \frac{5}{19} = 0.0921 \cong 9.21\%$$

⑥ And mean both No. will be multiply
$$P(R) \times P(B)$$

$$\frac{7}{20} \times \frac{5}{19} = 0.0921 \cong 9.20\%$$

Example:

what is the highest failure rate for a product if it is to have a probability of survival (ie (Successful operation) of 95% at 4000 Hrs? Assumed that the time to failure following an exponential distribution.

Solution:

$$\text{Reliability} = R(t) = e^{-\lambda t}$$

$$95\% = e^{-\lambda (4000)}$$

$$\text{failure rate} = \lambda = \frac{1}{\theta}$$

$$5\% = \lambda = \frac{\theta}{4000}$$

$$\boxed{\theta = 200}$$

Log	ln
$\log_{10}$	Lne

$\ln e = 1$

$\ln \rightarrow$ Natural log.

Solution:.

$$R(t) = e^{-\lambda t}$$

$$\frac{95}{100} = e^{-[\lambda \times 4000]}$$

$$0.95 = e^{-\lambda \times 4000}$$

$$\lambda = 12.8/10^6 \text{ hrs.}$$

$$\lambda = 0.0000128/\text{hrs.}$$

Thus the highest failure rate is $12.8/10^6$ hours for a riability of 0.95 @ 4000 hrs.

②

$$0.95 = e^{-[4000\lambda]}$$

Taking ln on both side

$$\ln(0.95) = \ln e^{-[4000\lambda]}$$

$$\log a^b = b \log a.$$

$$-0.05129 = -4000\lambda \times \ln e$$

$$+0.05129 = +4000\lambda \times 1$$

$$\frac{0.05129}{4000} = \lambda$$

$$\lambda = 12.8 \times 10^{-6}$$

Example: 20 units are put on test &
run at their normal operating condition
for 1000 hours $\mathfrak{I}$ 6 of those units fail
at the following hours (550, 480, 6800, 780
860, 620). what is the mean time to
failure of the ~~for~~ product.

Solution:-

MTTF and MTBF are reflection of
the reliability of your product — θ (theta).

Failure rate $(\lambda) = \dfrac{No \cdot of \ Failure}{operating \ Time}$

MTTF & MTBF, $\theta = \dfrac{operating \ Time}{No \ of \ Failure}$

un Reliability = $\dfrac{6}{20} = 0.3$

the reliability = $1 - 0.3 = 0.70$

Reliability = 70%

Right data: Censor data lift

$$= \quad 10000$$

$$\theta \quad = \quad \frac{(1000 \times 14) + 550 + 420 + 680 + 750 + 860 + 62}{6}$$

$$= \quad 899.$$

$$\frac{17980}{6}$$

$$\theta \quad = 2996 \quad Hrs \ to \ failure$$

failure rate

$$\theta = \frac{1}{\lambda} \quad , \quad \lambda = \frac{1}{2996}$$

Failure rate $= \lambda = 0.00033$

$$MTTF \ \theta = \frac{1}{Failure \ rate}$$

$$= \frac{1}{0.00033} \quad =$$

$$\underline{MTTF \quad = \quad 2896 \ Hrs.}$$

$$\text{Slope} = \frac{\text{raise}}{\text{run}} = \frac{0}{\text{Raise}} = \text{\textemdash}$$

$$\text{Slope} = \frac{\text{raise}}{\text{run}} = \frac{\text{Run}}{0} = \infty$$

$y = mx$

CFD = constant failure distribution
exponential distribut

20/8/24.

You have collected on component and calculated that your product fit a weibull distribution.

$$Reliability = R(t) = e^{-\left(\frac{t}{\theta}\right)^{\beta}}$$

$$5000 = e^{-\left(\frac{5000}{8000}\right)^{2}} = e^{-0.3906}$$

$$R(5000) = 0.6766$$
$$= 67\% \ Reliability$$

$$\beta = 2$$
$$\theta = 8000$$
$$T = 5000$$

Inf. No. CFR = Constant failure Rate

Bath tube curve. Explain.

Final Exam 2024 (7).

Twenty components will CFR (constant failure Rate) was observed being used in a high Stressed environment. After 25 hours of use Saven. of them failure at time in hours.

2.1, 8.3, 10.9, 15.2, 16.3, 20.5
23.8

while the remaining is Still functioning calculate the following parameters.

① MTTF
② the failure rate
③ the time at which the reliability is 95%.
④ the Reliability of 50 hours e of use.

① .,

$$MTTF = \frac{(13 \times 25) + 2.1 + 8.3 + 10.9 + 15.2 + 16.3 \quad + 20.5 + 23.8}{7}$$

$$= \frac{(13 \times 25) + 97.1}{7} , \quad \frac{422.1}{7}$$

$$= \quad 60.3$$

② failure rate

$$\theta_2 \quad \frac{1}{\lambda}$$

$$= \frac{\phi}{\text{✗}} \quad \frac{1}{60.3}$$

$$= 0.0018$$

$$\lambda = 0.0165$$

$$\boxed{\lambda = 1.65}$$

③

$$\text{Reliability} = e^{\lambda t}$$

$$95\% = e^{0.0165t}$$

taking ln on both sides

$$Ln(0.95) = 0.0165t \, \ln e$$

$$-0.051 = -0.0165 \, t$$

$$t = \frac{0.051}{0.016}$$

$$\boxed{t = 3.20}$$

(4)

$$Reliability = 50 \, Hrs.$$

$$R(t) = e^{-\lambda t}$$

$$= e^{0.0165\left(50\right)}$$

$$= e^{0.85}$$

$$\boxed{R(t) = 2.28} \quad R(t) = 0.438$$

Problem (2024)(8)

Imported Problem solve by Teacher

① MTTF

It is the Sum of all test time divided by # of Failure.

$$MTTF = \frac{2.1 + 8.3 + 10.9 + 15.2 + 16.3 + 20.5 + [13 \times 25]}{7}$$

$$= \frac{422.1}{7} = 60.3 \text{ hours/Failure}$$

(ii) Failure rate

$$\lambda = \frac{1}{\theta} = \frac{1}{60.3}$$

$$\lambda = 0.0166 \frac{\text{Failure}}{\text{hrs.}}$$

(iii) time @ which Reliability is 85?

$$R(t) = e^{-\lambda t}$$

$$0.95 = e^{-0.0166 t}$$

$\ln 0.95 = \left[-0.0166\, t \right] \times 1$

$0.0513 = 0.0166\, t$

$t = \dfrac{0.0513}{0.0166}$

$$\boxed{t = 3.09 \text{ hours}}$$

(iv) Reliability after 50 hours of use

$$R(50) = e^{-0.0166 \times 50}$$

$\approx e^{(-0.0166 \times 50)}$

$$R(50) = 43.6\%$$

Probability:

which estimating that probability of
a complex system like Aircraft carrier
or an oil-Rig it is inpractical to
test those system to distribution .. so the
reliability of these system are calculated
by estimating the reliability of individual
components

Rule #1

If A & B are two events of
interest and P_A & P_B are then
respective probabilities of them occuring
then if A & B are independent

$$P(A \& B) = P_A \times P_B$$

Rule #2.

If A & B are mutually exclusive
the

or only one will be
available at
same
time

$$P(A \text{ or } B) = P_A + P_B$$

Rule #3

If $A_1, A_2, A_3 \ldots \ldots A_n$ are mutually exclusive event and they discribe all possible outcomes in a particular situation then.

$$P_1 + P_2 + P_3 + \ldots P_n = 1$$

Rule #4

If there are only two possible outcomes (Say) Success & failure

$$P(\text{success}) = 1 - P(\text{failure}).$$

Note

$$P(A) = \frac{\# \text{ of favourable outcomes}}{\text{Total possible} \# \text{ of outcome}}$$

So the reliability of event A occurring.

is $\qquad 0 < P(A) \leq 1$
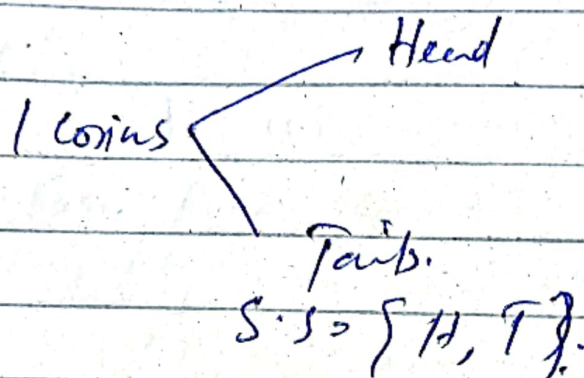
$\downarrow$
new.

$\downarrow$
Always
happen

Sample. ~~space~~ Space

A set of all possible outcomes that
can be occure

Q#01

1 coins

Head

Tails.

$$S \cdot S = \{ H, T \}.$$

Q#02
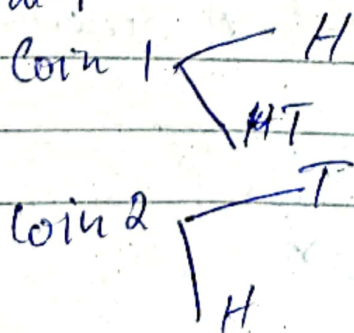
what 2t we flip a two coin what
are the possible outcome what acual
be sample spue

$$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}.$$

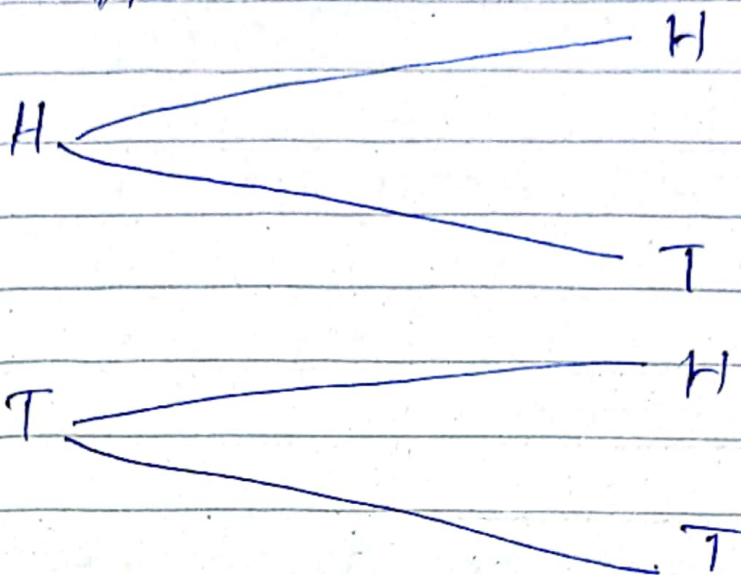$$S \cdot S = \{ H, T, T,H, T,T, H \cdot H \}.$$

Tree diagram

Coin 1

H

HT

Coin 2

T

H

coin #1   coin #2

H   H

                                       T

T   H

                                       T

$$= 2^n =$$

Probability
_____

3 Coins

Coin ①   Coin ②   Coins ③

H   H   H

              T   H

T   H   H

              T   T
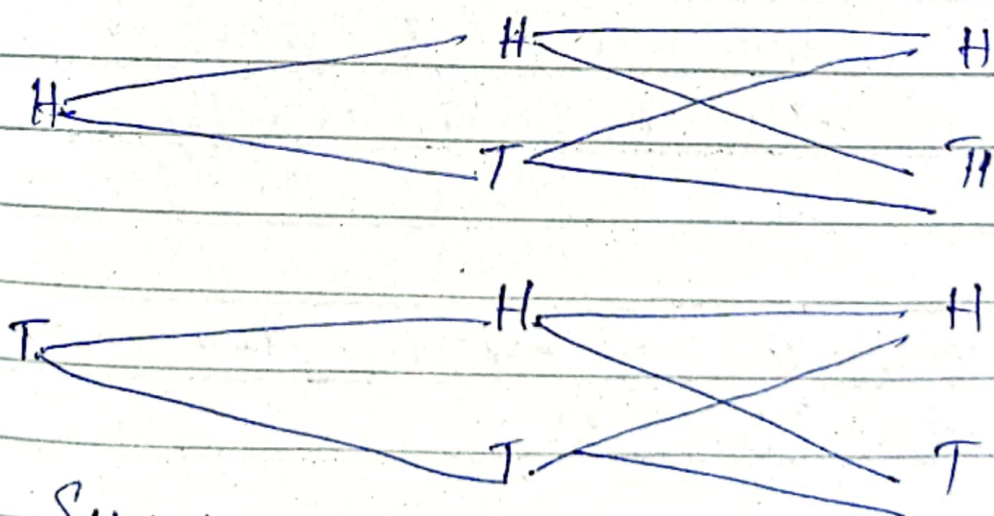
$\{ HHH, HH, H, HHT, HHT, HHT, T \}$

$$= 2^3, \quad 2 \times 2 \times 2 = 8$$

$\{ H,H, H, H H,T, H, T,H, H,T,T,$

$TH,H, THT, TTH, TTT \}.$

Probability. Question

If two fair coin a flip what
is Probability of getting at lee

(a) getting one Head

If three coin flip which
Probablh.
Atleil two tail.

(B) three coins are flap. what is.
what is the Probablty exactly one
Tail

also constructed of tree diagram
construct S.S &

(a) $\dfrac{7}{8}$ 2

(b) $\dfrac{3}{8}$

(c) 7/8.

Probability

Q N0:

A six sided die is tossed

(a) what is the probability of getting a "2"

(b) what is the probability of 3 or 5

(c) ~ ~ ~ . number that is

(d) almost 4

(d) N ~ ~ number that is. atmost greater then 3.

(e) what is probability of getting number that is less then. or equal to 5.

Solution:

(a) $P(2) = \frac{1}{6} = 0 \cdot 166$

(b) Probability of 3

$P(3or5) = \frac{2}{6} = \frac{1}{3}$

Probability of 5

$P(5) = \frac{1}{6}$

$$S.S = \{1, 2, 3, 4, 5, 6\}.$$

© Probability of almost 4.
$$\{1, 2, 3, 4\}.$$

$P(n \leqslant 4) = $ ~~[crossed out]~~ $\dfrac{4}{6} = \dfrac{2}{3} = 0.666$

ⓓ Probability of ~~almost~~ greater then 3.
$$S.S \{4, 5, 6\},$$

$P(n > 3) = $ ~~[crossed out]~~ $\dfrac{8}{6} = \dfrac{1}{2} = 0.5 =$

ⓔ less then equal 5 $S.S \{1, 2, 3, 4, 5\}$.
$P(n \leqslant 5)$ ~~[crossed out]~~ $= \dfrac{5}{6} = 0.83.$

2

Q.No

A Jar contains 7 red marbles.
6 green marbles, 5 blue marble
and 2 yellow marbles

① what is the probability of selecting
green marbles.

②   N   (N   (r

blue marbles.

③ Green or yellow
④ Red then blue with replacement
⑤ Red then blue without replacement
⑥ Red & blue without replacement
⑦ what is the probability of selecting
red then green marble or getting
a blue marble and then yellow
marble with replacement

then → multiply
are → Sum

thow → multiply are → Sum	Foot Ball → placed again placed in same basket without replacement Ball pick & place in other basket & then take new..

Ⓐ

$P(Red) = \dfrac{7}{20} = 0.35$

Ⓐ S.S = $\{7Red, 6q, 5B, 2y\}$

Total marble = 20

$= \dfrac{7}{20} = 0.35$

Ⓑ $P(blue) = \dfrac{5}{20} = 0.25$

Ⓒ $= \dfrac{8}{20} = 0.4$

Ⓒ

Ⓓ $\dfrac{12\$}{20} =$

$d = 5 \times 7 = 35$

red $\dfrac{7\text{@}}{20} = 0.6$ 0.35

blue $\dfrac{5}{20} = 0.4$

e $\dfrac{7}{20} = 0.35$

Ⓔ $\dfrac{5}{19} = 0.263$

f) $\dfrac{17}{20} = 0.6$

$\dfrac{12}{20} = 0.63$

$5/20 \quad \dfrac{7}{20}$

$\dfrac{2}{20} \quad \dfrac{6}{20}$

or =

Solve by Weather:

Color	marble Qty.
Red (R)	7
Green (G)	6
Blue (B)	5
Yellow (Y)	2
Total	20

(a) $\quad P(G) = \dfrac{6}{20} = \dfrac{3}{10} = 0.30 = 30\%$

Not Selecting the green marble

$$= 1 - 0.3 = 0.7 = 70\%$$

(b)

$\quad P(B): \dfrac{5}{20} = \dfrac{1}{4} = 0.25 = 25\%$
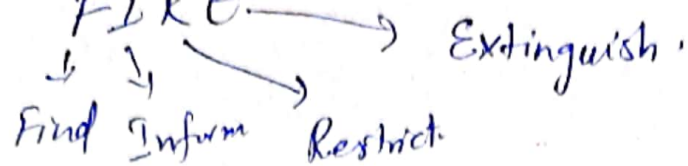
(c) $\qquad\qquad OR = $ Adding both Probability

$P(G) + P(Y).$

$$\dfrac{6}{20} + \dfrac{2}{20} = \dfrac{8}{20} = \dfrac{2}{5} = 0.4 = 40\%$$

There is a 40% chance of selecting a B or Y marbles.

FIRE ——→ Extinguish.

Find Inform Restrict

(d) Red then blue with replacement
it means once you take out red
marble on 1st try you put it back
for 2nd tray.

$$P(R) \cdot P(B).$$

$$\frac{7}{20} \cdot \frac{5}{20} = 0.0875 = 8.75\%.$$

(e) Red then blue without replacement it mean
once you take out red marble on 1st try you
put separate (other basket) and then try.

$$P(R) \cdot P(b)$$

$$= \frac{7}{20} \cdot \frac{5}{19} = 0.0921 = 9.21\%.$$

(f) And mean both no: which be addon multiply
$$P(R) \times P(B)$$

$$= \frac{7}{20} \times \frac{5}{19} = 0.6 = 60\%$$

$$= 0.35 \times 0.263 = \frac{0.0921}{} = $$

$$= 9.20\%.$$

(f)

Red and Blue without : Replacement

[R][B] OR [B][R]

$[\frac{7}{20}][\frac{5}{19}] + [\frac{5}{20}][\frac{7}{19}]$

$(0.35)(0.2632) + (0.25)(0.3684)$

$0.0921 + 0.0921$

$= 0.1842 = 18.42\%$

(G)

[R][G] or [B][Y]

$[\frac{7}{20}][\frac{6}{20}] + [\frac{5}{20}][\frac{2}{20}]$

$(0.35)(0.30) + (0.25)(0.1)$

$0.105 + 0.025 = 0.13 = 13\%$

$= 13\%$

Mutually exclusive: never come exclusiure simultaneously.

Note:-

* Dice and coin throwing. are always. independent event

* they don't have any past memory.
* they are unbiased. and are all equally probable & can occuare randomly.

Adding Rule:

$$P(A \text{ or } B) = P(A \cup B).$$

occuring together

$$= P(A) + P(B) - P(A) \& (B) \{ P_A \cap P_B \}$$
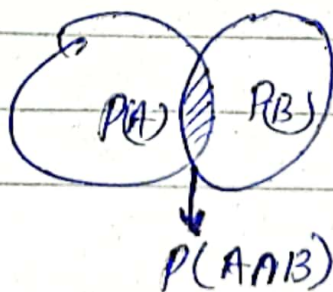
+ two events which are **mutually. Exclusive.**

(the cannot occur simultaneously)

$$P(A \& B) = 0$$

∴ A & B are different Events.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

thugh Venn diagram

$$P(A \cap B).$$

Multiplication Rule

- Conditional Probability (event are not independent.

$$P(A/B) = \frac{P(A \& B)}{P(B)}$$

$\downarrow$

Probability of A given (B)

Multiply Both side by $P(B)$.

$$P(A/B) \times P(B) = \frac{P(A \& B)}{P(B)} \times P(B)$$

$$P(A \& B) = P(B) \times P(A/B)$$

$$P(B/A) = \frac{P(A \& B)}{P(A)}$$

Multiply by both side by $P(A)$.

$$P(A \& B) = P(B/A) \times P(A)$$

Independent Event (IE)

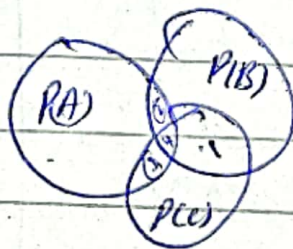An I-E is that if one event does not depend on another event.

For IE

$$P(A/B) = P(A).$$

$$P(B/A) = P(B).$$

Additional Rules

Apply add Rule to 3 or more Probability

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) + P(A \cap B)$$

$$+ P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)$$

Q No

what is the probability of rolly a 4 or
3 when a six side choice is tossed
up.

Sol

$P(A \; OR \; B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(4) + P(3) - P(4 \cap 3).$

$P = (1, 2, 3, 4, 5, 6).$

$= 7 - P($

$P(A) = \dfrac{1}{6} + \dfrac{1}{6}$

$= \dfrac{2}{6}$

$P(A \cup B) = \dfrac{2}{6} =$

$P(A \cup B) = \dfrac{1}{3}$

Slv.

$$P[3 \cup 4] =$$

$$P(3) = \frac{1}{6}$$

$$P(4) = \frac{1}{6}$$

$$P(3 \cap 4) = 0$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

$$= \frac{1}{6} + \frac{1}{6} - 0$$

$$= \frac{2}{6}$$

$$P(A \cup B) = \frac{1}{3}$$

Q.40:

A Bag consist of 8 Red ball marbles.
7 blue marble, 6 green marbles. and
4 yellow marbles.
What is the probability of
Selecting.

ⓐ A red marble

ⓑ A Blue marble
    first try and then a green marble
    on second try with replacement

ⓒ Two yellow marble on the first try & then.
   a red marble on the second try without repla

ⓓ two blue marbles with replacement

ⓔ two green marble without replacement

Sol.

Color	Qty of marble
Red	8
Blue	7
Green	6
Yellow	4
	25 marble

(a) Probability of Red marble

$$P(R) = \frac{8}{25} = 0.32 = 32\%$$

32% chance of selecting a red marble on first try.

(b)
$$P(B) = \frac{7}{25} = 0.28$$

with replacement Green
$$P(G) = \frac{6}{25} = 0.24$$

$$P(BG) = \frac{7}{25} \times \frac{6}{25} = 0.067 = 6.72\%$$

(b)
$$P(BG) = \frac{7}{25} \times \frac{6}{25}$$
$$= 0.067$$
$$P(BG) = 6.72\%$$

(c)
$$P(Y) = \frac{4}{25} = 0.16$$

without replacement Red.

$$P(R) = \frac{8}{24} = 0.333$$

(c)
$$P(YR) = \frac{4}{25} \times \frac{8}{24} =$$
$$= 0.16 \times 0.333$$
$$= 0.053$$
$$= 5.3\%$$

(d) two blue marble with replacement
$$= ;$$

$$\boxed{P(A \cup B = \frac{P(B) + P(B) - P(B \cap B).}{0.28 + 0.28 - (5).}}$$

$$\frac{7}{25} \times \frac{7}{25}$$

$$P(BB) = 0.28 \times 0.28 = 0.078 = 7.84\%$$

© Two Green marble without replement

$$= \frac{6}{25} \times \frac{6}{24}$$

$$= 0.24 \times 0.25 = 0.06 = 6\%$$

$$= 0.059$$

$$= \frac{6}{25} \times \frac{3}{24}$$

$$0.24 \times 0.203$$

$$P(GG) = 0.05 = 5\%$$

Q.No:-

the Reliability of a missile is 0.85. If a Salvo of two missile of is fire.

what is the probability of at least one hit

Note:

Assume independance of missile hits

Solve the Problem using

(a) Probability Equation for two independent event

(b) Sequential tree Diagram Construction

(c) Basic Probability technique

Hint:

Probability of A or B occurring if A & B are independent.

$$P(A+B) = P(A) + P(B) - P(A)P(B)$$

$P(missile \cdot 1) = P(m_1) = 0.85.$

$P(missile\ 2) = P(m_2) = 0.85$

Solv (Teacher).

Let

A be the event for 1st missile hit

B be event for 2nd missile hit

So

$P(A) = P(B) = 0.85$

failure). $P(\bar{A}) = P(\bar{B}) = 0.85$

$P(A+B) = P(A) + P(B) - P(A)\,P(B).$

$= (0.85) + (0.85) - P(0.85)(0.85).$

$P(A+B) = 0.9775$ Ans

$P(A+B) = 98\%.$

ⓑ S.S = { HH, HM, MH, MM }.

there are 4 possible outcomes.

{ AB, A B̄, Ā, B, Ā B̄ }.

for Possibility of alleast 1 hit   X

$=$

$= \dfrac{3}{4} = 0.75 = 75\%$

Now lets take the probability
of both missile.

$$P(A) \times P(B) = P(\bar{A} \bar{B})$$

$$[0.15] \times [0.15].$$

$$P(\bar{A} \cdot \bar{B}) = 0.0225 = 2.25\%$$

Probability. alleast one hit

$$P(S) = 1 - P(f)$$

$$= 1 - 0.0225$$

$$P(S) = 0.9975$$

$$P(S) = 97.75\%$$

*. Oil Pumpss - Quality Reports
— oil loses.

⑥ Sequential Tree diagram.

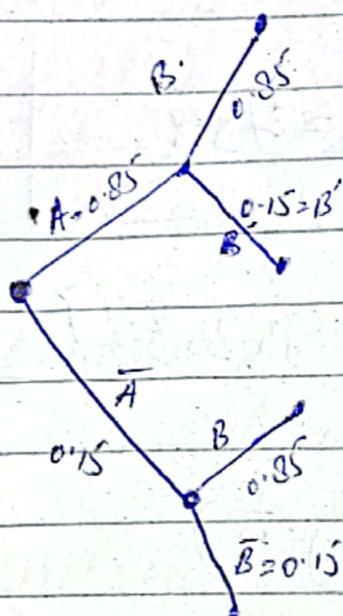$\{AB, A\bar{B}, \bar{A}B, \bar{A}\bar{B}\}$ Rough.
0.7225 0.1275 0.1275 0.0225   Work

$= 0.7225 + 0.1275 + 0.1275$

$= 0.9775.$

B $\cdot$ 0.85
A - 0.85
$0.15 = \bar{B}$
B

$\bar{A}$
0.15   B   0.85
$\bar{B} = 0.15$

$= 1 - 0.9775$

$= 0.0225$ miss

M.E = mutually exclusive

$P(AB) = 0.85 \times 0.85 = 0.7225$

$P(A\bar{B}) = 0.85 \times 0.15 = 0.1275$

$P(\bar{A}B) = 0.15 \times 0.85 = 0.1275$

$P(\bar{A}\bar{B}) = 0.15 \times 0.15 = 0.0225$

Probability of hit divided by.
Summing the product of each.
path which leads to atleast
on hit As each path are (M.E)

$P(AB) + P(A\bar{B}) + P(\bar{A}B)$
$= 0.7225 + 0.1275 + 0.1275$

$\boxed{P(System) = 0.9775}$

Note:
$P_1 + P_2 + P_3 + P_4 + \text{------} P_n = 1$
For

Q.No

Sarah is deciding which course she wants to take in her next college semester. Probability that she enrolls in an Algebra course is 0.3 and Probability that she enrolls in a biology course is 0.7. Probability that she will enroll in Algebra course given that she will enrolls in biology course is 0.4

(a) What is the Probability that she will enroll in both an Algebra & Biology course

(b) What is the Probability that she will enroll in an algebra course or a biology course

(c) Are the two events independent (Proof that).

(d) Are the two events mutually exclusive.

$$P(A \& B) = P(B/A) \times P(A)$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

(a).

$P(A)$ = Algebra = 0.3.
$P(B)$ = biology = 0.7
$P(A \cap B)$ = 0.4

$P(A)$ = $\dfrac{0.3}{1}$ = 0.3   Rough work

$P(A \& B)$ = (0.4)(0.3).
= 0.12

Sol. (Teacher).

$P(A)$ = 0.3
$P(B)$ = 0.7
$P(A/B)$ = 0.4

(a)

$$P(A \& B) = P(A/B) \times P_B$$

= 0.4 × 0.7

$P(A \& B)$ = 0.28 = 28%

(b)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

$$= 0.3 + 0.7 - 0.28$$

$$= \cancel{0.72}$$

$$P(A \text{ or } B) = 0.72 = 72\%$$

(c)

Are that two events independent

$$P(A/B) = P(A).$$

$$0.4 \quad \geq \quad 0.3.$$

$$0.4 \quad \neq \quad 0.3.$$

$$\downarrow \text{ independent}$$

Events are not ~~dependent~~

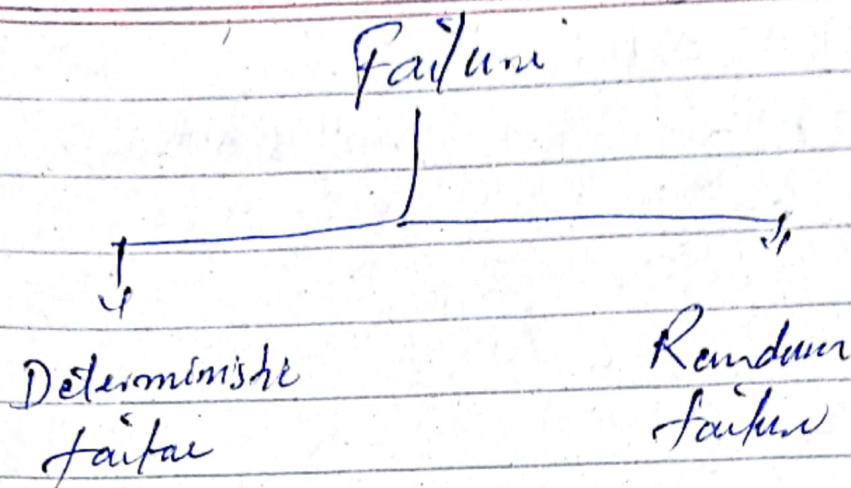(d) Are the two events mutually Exclusive we use

$$P(A \text{ and } B) = 0$$

But we know that

$$P(A \text{ and } B) \neq 0$$

$P(A \& B) = 0.28$

Event $(A \& B)$ are not mutually.
Exclusive event.

Failure

```
            Failure
              |
     ┌────────┴────────┐
     ↓                 ↓
Deterministic      Random
failure            failure
```

Random failure:

They may exhabit a Pattern that can be modelled by some Probability Distribution

Random Variable:-
They allow us to pass from experimental outcome to as Numerical function of the outcome.

Types of random variable
(continous unknow).
① descreate random variable
② Continous random variable

Standard deviation

Simple portion %

(Temp, man, at heat)

FMEA

Failure mode & effective analysis.

Design
failure
Mode & Effection
analysis.

(DFMEA)

Process.
failure
Mode & effection
analysis.

(PFMEA)

mid term Paper / pattern

// MTTA vs MTBF

Q.

- $R(t) = 1 - fail$

Note the Diffference b/w following

- Quality vs Releable.
  MTTF vs MTBF
- Name potinal ways to improve
  relebility.

- why product fail.

Q.
  Do unt one put on test.

MTTF,
calculet the faile rate

Q. both find cumm. Sketch
  Expon the three main failure
  rate Feature
  $$f(t) = \begin{cases} 0.002 e^{-0.002t} & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) $R(t) =$ (c) median
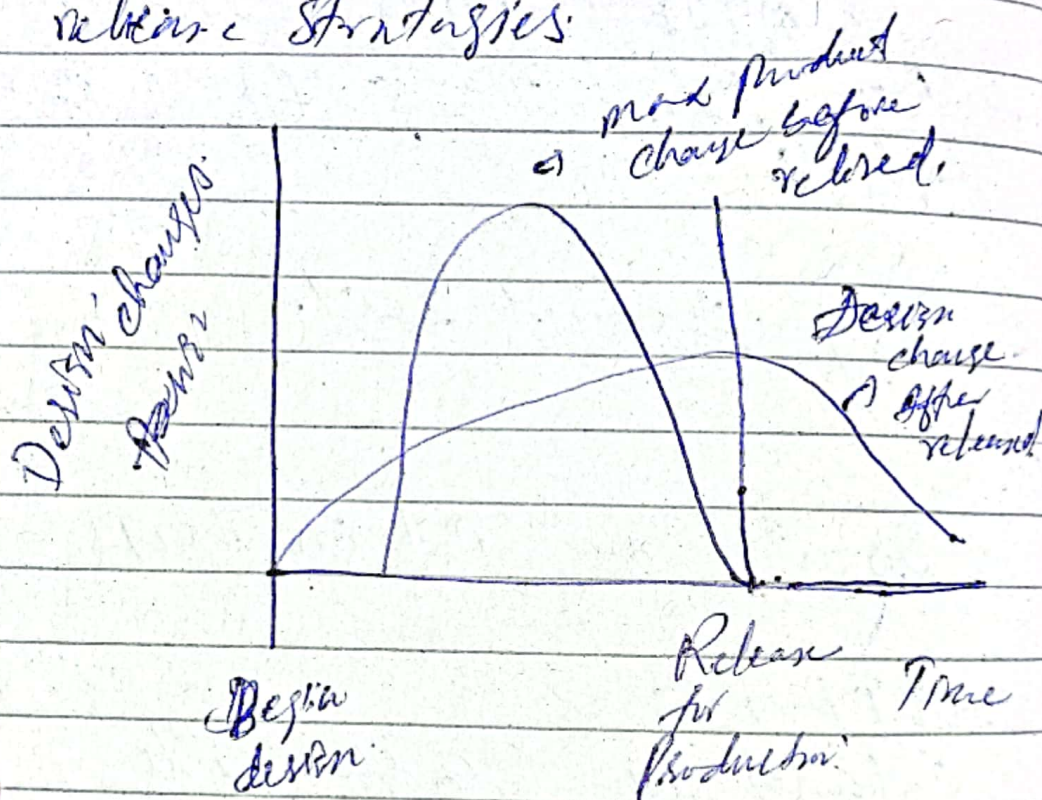(b) MTTF

Proof mathematical MTTF -

$$MTTF = \int_0^\infty R(t)\, dt$$

Details

D. release Strategies

Design changes
Rate

a) max product change before released.

Design change after released.

Begin design

Release for Production.

Time

Mid life upgradation (MLU)

• FMEA :-

- Technique Risk Assessment prior to release of design, process or service
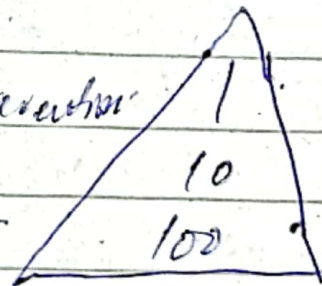
Severity
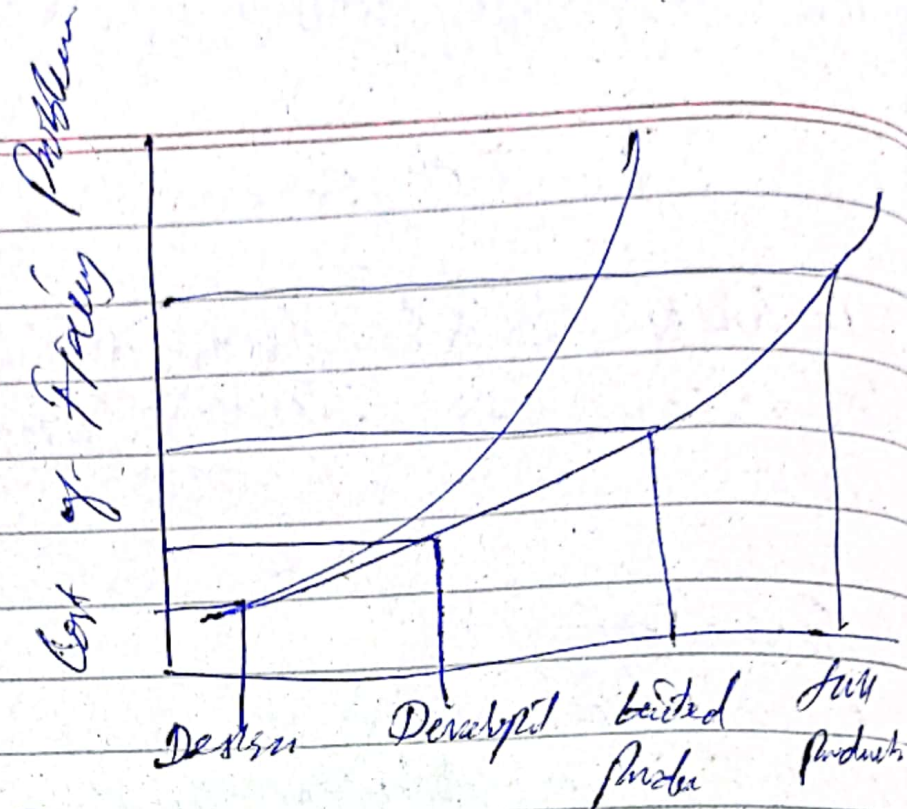occurance
Probability of detection in the event of occurrence.

$$RPN \rightarrow Number \rightarrow S \times O \times D.$$

Risk Priority

$$10 \times 10 \times 10$$

the

1 — 10 — 100 Rule

one dollar spent on prevention
10 dollar on correction.
100 dollar on failure cost

1
10
100

Cost of Status Problem (y-axis)

Design — Development — Limited Production — full Production (x-axis)

Stage ①

① Cost of design fixation is less. Cost of Development is more cost as compared to design. Stage ②

Stage ③

Limited production cost is high. at this stage and farther at this stage the fixation cost of very high the they design. the production machines for this product.

Stage ④

full production the fixture at machine design.

Dies, fixture has produced machines are design without assembly has design.

CQI = continous Quality - 1) High
Viscosity

Ball rotation

Befrits of PMSA.

1) Higher reliability
2) Better quality
3) increased Safety,
4) Enhanced customer stff' Satisfation
~~contribtn~~
5) Contributes to cost saving
6) Decreasen development

DPMEA : Roadmap. (Note
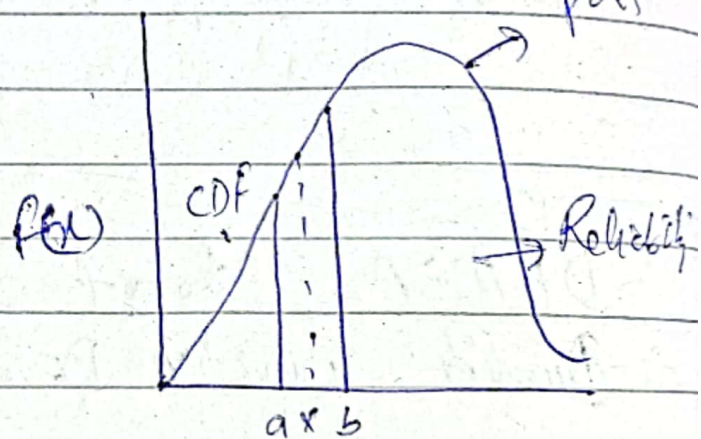Quality Function Development :

— Random variable.

Redundancy → backup → Standby.

— Discreet
— its Continous

① Probability Density Function (PDF)

f(x)   CDF   pdf   → Reliability

a & b
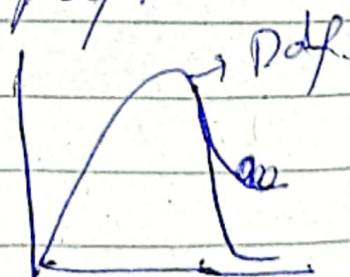
② Cumulative   Function.
③ Reliability function. @ R(t)

Probabilty Density Function (Pdf).

for f(x) to be a pdf.

Exponential Reliability

$$S(\infty) + \infty = \infty$$
$$\alpha \cdot S(\infty) = \infty$$
$$\frac{1}{\alpha} = 0$$

Example (pdf)

$$f(n) = \begin{cases} c(4n - 2n^2), & 0 < n < 2? \\ 0, & \text{otherwise} \end{cases}$$

$$f(n) = \int_0^2 4n - 2n^2$$

$$f(n) = 4 + 4n.$$
Apply the limit.
$$f(n) + 0$$
$$f($$

$$\int_{-\infty}^{\infty} f(n)\, dn = \int_{-\infty}^{0} f(n)\, dn + \int_{0}^{2} f(n)\, dn + \int_{2}^{\infty} f(n)\, dn = 1$$

Sol

$$= \int_0^2 c(4n - 2n^2)\, dn = 1$$

Example:

Time to failure (in hr) for a capacitor has the following pdf what is the probability of failure by $t = 200$ hr?

$$f(t) = 0.01 e^{0.001}$$

Since its time it must

$$f(x) = \int_{-\infty}^{n} f(n) \, dn.$$

$$= \int_{0}^{200} 0.01 e^{0.001} f(t) \, dt$$

$$P(t \leq 200) = \int_{0}^{200} 0.01 e^{-0.001 t} \, dt$$

$$F(n) = \int_{-\infty}^{n} f(y)\, dy$$

$$R(n) = \int_{x}^{\infty} f(y)\, dn$$

$$f(n) = \frac{dF(n)}{dn}$$

$$F(x) = 1 - R(n).$$

$$: \frac{dF(n)}{dn} = \frac{d \, e \, R(n)}{d(n)} = f(n).$$

Why Improve Reliability

Ans. potential ways to Improve Reliability

Failure rate or Hazard.

$$\lambda = \frac{f(t)}{R(t)} \quad \text{Reliability}$$

function

Ron.

Prone

$$1 - R(t) = \frac{dF(t)}{dt}$$

$$\lambda(t) = \frac{f(t)}{Rt}$$

$$\frac{\frac{dF(t)}{dt}}{R(t)} = \frac{\frac{d[1-R(t)]}{dt}}{R(t)} = \frac{-\frac{dR(t)}{dt}}{R(t)}$$

$$\lambda(t)\,dt = \frac{dR(t)}{R(t)}$$

$$\int_0^t \lambda(t)\,dt = \int_1^{R(t)} \frac{dR(t)}{R(t)} = -\ln R(t)$$

$$R(t) = e^{-\int_0^t \lambda(t)\,dt}$$

Problem :-

Given the Hazard function when t is measured in operating H hours. what is the design life if a 0.98 reliability is designed. Take Hazard function.

$$\lambda(t) = 5 \times 10^{-6} (t)$$

Sol:-

$$R(t) = e^{-\int_0^t \lambda(t)\,dt}$$

$$= e^{-\int_0^t 5 \times 10^{-6} t\,dt}$$

$$R(t) = e^{-\left[\frac{5 \times 10^{-6} \times t^2}{2}\right]_0^t}$$

$$0.98 = e^{-\left[2.5 \times 10^{-6} t^2\right]}$$

$$\ln 0.98 = \ln e^{-\left[2.5 \times 10^{-6} t^2\right]}$$

$$\ln 0.98 = 2.5 \times 10^{-6} t^2 \times 1$$

$$t^2 = \frac{0.0202}{-2.5 \times 10^{-6}}$$

$$\boxed{t = 89.88 \text{ hrs}}$$

$$f(x) = (pdf)$$
$$F(x) = CDF$$
$$(CT)' = pdf$$

Problem:.

pdf

Given that $f(n) = 0.048 \, n [5-n]$

(a) Verify that f is a pdf
(b) What is the probability that "x" is greater than 4
(c) What is the probability that n is in b/w 1 & 3 inclusive inclusive.

Sol.

$$\int_{-\infty}^{+\infty} f(n) \, dn = 1 \longrightarrow \boxed{Formula}$$

↓

integral from $-\infty$ to $+\infty$ for any pdf $= 1$

Sol.

$$\int_{-\infty}^{+\infty} f(n) \, dn = 1$$

(a)

$$f(x) = 0.048x(5-x)$$

for

$$\int_{-\infty}^{+\infty} 0.048x(5-x)$$

(a) If

$$f(x) = 0$$

$$0.048(0)[5-0]$$

$$f(x) = 0$$

(b)

If $f(x) = 5$

$$= 0.048(5)[5-5]$$
$$f(x) = 0$$

So x must be $4w$ $0 \leq x \leq 5$

If It will give the result
So if we integrate from.

$$\int_{0}^{5} f(x) \, dx = 1$$

$$= \int_{0}^{5} 0.045(x)\,(5-x)\,dx$$

$$\Rightarrow \text{ So } f(x) \text{ is.} \quad < \text{Pdf.}$$

(b) $p(4 \leq x < 5) = \int_{4}^{5} 0.048\,x\,(5-x)\,dx$

$P(4 \leq x \leq 5 = 0.104 = 10.4\%$

(c)

$$P(1 \leq x \, 3) = \int_{1}^{3} 0.048\,x\,(5-x)\,dx.$$

54.4%

Design Life

a

$\uparrow_{q0}$ Reliability

pdf
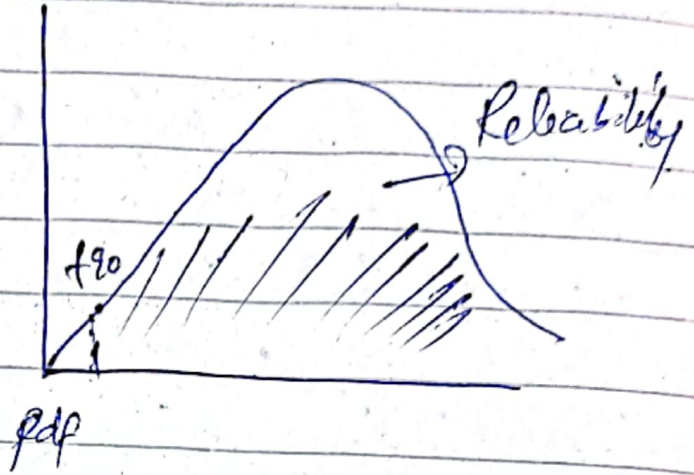
Probabili:

Problem:

Given the following pdf. for the random variable T the time in operating hr) to failure of a compressor. what is the its reliability for a 100 hr. operating life? what is the design life for 95% reliability.

Solution

$$f(t) = \begin{cases} \dfrac{0.001}{(0.001t+1)^2} & t \geq 0 \\[2mm] 0 \end{cases}$$

$$f(t) = \dfrac{0.001}{(0.001t+1)^2} \qquad t \geq 0$$

$$R(t) = \int_t^{\infty} \dfrac{0.001}{(0.001t+1)} \, dt$$

let

$$U = \left[0.001\,t + 1\right]$$

$$\dfrac{dU}{dt} = 0.001$$

$$\frac{du}{0.001} = dt$$

$$= \frac{0.001}{0.001} \int_{t}^{v} \frac{du}{0.001\,U^2}$$

$$= \int_{t}^{v} \frac{dU}{(U)^2}$$

$$= \int_{t}^{v} U^{-2}\,dU$$

$$= \left\{ \left[\frac{U^{-2+1}}{-2+1} \right]_{t}^{v} \right.$$

$$= -\left[\frac{1}{U} \right]_{t}^{v} \left. \right\}$$

$$\left[-\frac{1}{2} + \frac{1}{U} \right]$$

$$0 + \frac{1}{U}$$

$$O \to \frac{1}{U}$$

$$R(t) = \frac{1}{(0.001t + 1)}$$

$$0.95 = \frac{1}{(0.001t + 1)}$$

$$(0.001t + 1) \; 0.95 = 1$$

$$R(100) = \left[\frac{1}{[0.001 \times 100 + 1]} \right]$$

$$\boxed{R(100) = 0.909}$$

(b) Design life for 95% reliability

$$0.95 = \frac{1}{0.001t + 1}$$

$$0.95 \times (0.001 \, t + 1) = 1$$

$$0.00095t + 0.95 = 1$$

$$0.00095t = 1 - 0.95$$

$$t = \frac{0.05}{0.00095}$$

$$\boxed{t = 55.55}$$

$$\boxed{t = 52.631 \text{ hrs.}}$$

Median

Exact median of
an function = 50th percentage.

Problem:

Consider the following pdf with
x in hrs.

$$f(x) = \begin{cases} \frac{3}{8}(4x - 2x^2), & 0 < x < 2 \\ 0 \end{cases}$$

other wise median. find median

$$P(x) = R(x) = 0.5$$

$$P(x) = \int_0^x \frac{3}{4}(4t - 2t^2) \, dt = 0.5$$

$$F(x) = \frac{3}{8}\left(2x^2 - \frac{2}{3}x^3\right) = 0.5$$

$$F(t_{50}) = \frac{1}{2}$$

$$f(x) = R(x) = 0.5$$

$$= \int_0^x \frac{3}{8}\left[4t - 2t^2\right] dt = \frac{1}{2}$$

$$\frac{3}{8}\left[2t^2 - \frac{2}{3}t^3\right]_0^x = \frac{1}{2}$$

$$2x^2 - \frac{2}{x}x^3 = \frac{8}{6}$$

$$-\frac{2}{3}x^3 + 2x^2 = \frac{4}{3}$$

$$= 2.736 \quad \times \quad 0 < x < 2$$
$$= 0.99585$$
$$= -0.731 \quad \times$$

$$0 \quad \therefore$$